

Definition 0.0.1 (expectation). Let X be a r.v. on a probability space $(\Omega, \mathcal{A}, \mathbb{P})$. The **expectation** of X is

$$\mathbb{E}(X) = \int_{\Omega} X(\omega) d\mathbb{P}(\omega).$$

When X is a simple random variable, i.e. it takes on only finitely many values, it can be written as

$$X = \sum_{i=1}^n x_i \mathbf{1}_{A_i} \iff X(\omega) = \sum_{i=1}^n x_i \mathbf{1}_{A_i}(\omega), \quad \omega \in \Omega$$

where $x_i \in \mathbb{R}$ and $A_i = \{\omega : X(\omega) = x_i\} \in \mathcal{A}$. Its expectation is given by

$$\mathbb{E}(X) = \sum_{i=1}^n x_i \mathbb{P}(A_i) = \sum_{i=1}^n x_i \mathbb{P}\{X = x_i\}.$$