Theorem 0.0.1 (Ito's Lemma, 1D)**.** *Suppose that $g : [0,T]\times\mathbb{R} \to \mathbb{R}$ is a function of class $C^{2,1}([0,T]\times\mathbb{R},\mathbb{R})$, i.e. g is twice continuously differentiable. Then for any Ito process X, the process $Y_t = g(t, X_t)$, $t \in [0,T]$, is an Ito process (and therefore also a continuous semimartingale). Moreover, its canonical decomposition is given by the Ito formula*

$$dg(t, X_t) = g_t(t, X_t)\,dt + g_x(t, X_t)\,dX_t + \frac{1}{2}g_{xx}(t, X_t)\,d\langle X\rangle_t.$$

or equivalently,

$$g(t, X_t) = g(0, X_0) + \int_0^t g_t(s, X_s)\,ds + \int_0^t g_x(s, X_s)\,dX_s + \frac{1}{2}\int_0^t g_{xx}(s, X_s)\,d\langle X\rangle_s.$$

If $dX_t = \alpha_t\,dt + \beta_t\,dW_t$, then

$$dg(t, X_t) = g_t(t, X_t)\,dt + g_x(t, X_t)\alpha_t\,dt + g_x(t, X_t)\beta_t\,dW_t + \frac{1}{2}g_{xx}(t, X_t)\beta_t^2\,dt.$$