

**Lemma 0.0.1** (conditional expectation process is a martingale). *Let  $X$  be an  $\mathcal{F}_T$ -measurable and integrable random variable. Then the process defined by  $M_t = \mathbb{E}(X|\mathcal{F}_t)$  for  $t \in [0, T]$  is an  $\mathcal{F}$ -martingale.*