Definition 0.0.1 (martingales, submartingales, supermartingales)**.** Consider a real-valued, $\mathcal{F}$-adapted process $M = (M_t)_{t\in[0,T]}$, defined on a filtered probability space $(\Omega, \mathcal{F}, \mathbb{P})$. If

(i) M is integrable, that is, $\mathbb{E}|M_t| < \infty$ for $t \in [0,T]$, and

(ii) (martingale property) for any $0 \leq s \leq t \leq T$,

- $\mathbb{E}(M_t|\mathcal{F}_s) = M_s$, then M is an $\mathcal{F}$-**martingale**. The expectation is a constant: $\mathbb{E}(M_t) = \mathbb{E}(M_0)$, for all $t \in [0,T]$.
- $\mathbb{E}(M_t|\mathcal{F}_s) \geq M_s$, then M is an $\mathcal{F}$-**submartingale**. The expectation is increasing: $\mathbb{E}(M_t) \geq \mathbb{E}(M_0)$ for any $t \in [0,T]$.
- $\mathbb{E}(M_t|\mathcal{F}_s) \leq M_s$, then M is an $\mathcal{F}$-**supermartingale**. The expectation is decreasing: $\mathbb{E}(M_t) \leq \mathbb{E}(M_0)$ for any $t \in [0,T]$.