Theorem 0.0.1. *Let X be a square integrable random variable, i.e. $\mathbb{E}\left[X^2\right] < \infty$. Then $\mathbb{E}[X|\mathcal{G}]$ is the orthogonal projection of X on $L^2(\Omega, \mathbb{P})$. This means that for every square integrable $\mathcal{G}$-measurable Z,*

$$\mathbb{E}\left[(X - Z)^2\right] \geqslant \mathbb{E}\left[(X - \mathbb{E}[X|\mathcal{G}])^2\right]$$

with equality if and only if $Z = \mathbb{E}[X|\mathcal{G}]$.