Proof. (ii) Assume that $M_{t_0} \neq 0$ for some $t_0 \in [0, T]$. Then $\mathbb{E}(M_{t_0}) > 0 = \mathbb{E}(M_0)$, which contradicts the property that M is a supermartingale.

(iv) We may assume, without loss of generality, that M is a continuous bounded martingale and $M_0 = 0$. This is because one can always take $M' = M - M_0$ and stop the continuous local martingale M' such that it is bounded. Then

$$\mathbb{E}[M_t^2 - \langle M \rangle_t] = M_0^2 - \langle M \rangle_0 = 0 \implies \mathbb{E}\left[M_t^2\right] = \mathbb{E}[\langle M \rangle_t] = 0$$

for any $t \in [0, T]$ and thus $M_t = 0$ for any $t \in [0, T]$.

(v) It suffices to apply (iv) and to observe that the assumption that sample paths of M are continuous functions of finite variation implies that the quadratic variation of M vanishes. ∎