Theorem 0.0.1 (Cauchy–Schwarz inequality)**.**

1. *(definite integrals) Let f and g be real functions which are continuous on the closed interval $[a, b]$. Then:*

$$\left(\int_a^b f(x)g(x)\,dx\right)^2 \leq \int_a^b f^2(x)\,dx \int_a^b g^2(x)\,dx.$$

As a corollary, we have

$$\left(\int_a^b f(x)\,dx\right)^2 \leq (b-a)\int_a^b f^2(x)\,dx.$$

2. *(expectations) For any two random variables X and Y,*

$$|\mathbb{E}(XY)|^2 \leq \mathbb{E}(X^2)\mathbb{E}(Y^2),$$

or equivalently,

$$|\mathbb{E}(XY)| \leq \sqrt{\mathbb{E}(X^2)\mathbb{E}(Y^2)}.$$