Definition 0.0.1 (conditional expectation w.r.t. a partition)**.** For any partition $\mathcal{D} = \{D_1, \cdots, D_n\}$ of Ω, the conditional expectation of a simple random variable X is given by

$$\mathbb{E}(X|\mathcal{D}) = \sum_{i=1}^{n} \mathbb{E}(X|D_i)\mathbf{1}_{D_i} = \underbrace{\sum_{i=1}^{n}\sum_{j=1}^{m} x_j \mathbb{P}(D_j^X|D_i)\mathbf{1}_{D_i}}_{\text{a simple } \sigma(\mathcal{D})\text{-measurable r.v.}},$$

where $\mathbb{E}(X|D_i) = \mathbb{E}(X\mathbf{1}_{D_i})\mathbb{P}(D_i)^{-1}$ is the usual conditional expectation, and $\mathcal{D}(X)$ partitions Ω w.r.t. X.

If we have two simple random variables X and Y, then

$$\mathbb{E}(X|Y) := \mathbb{E}[X|\mathcal{D}(Y)] = g(Y)$$

for some measurable function g, which, clearly, is $\sigma(Y)$-measurable.