Theorem 0.0.1 (Doob's optional sampling theorem). *Let $\sigma \leq \tau$ be two stopping times taking values in $\{0, 1, \ldots, N\}$. For any martingale, submartingale or supermartingale X, we have*

(i) The random variables X_σ and X_τ are integrable.

(ii) $\mathbb{E}[X_\tau|\mathcal{F}_\sigma] = X_\sigma$ (if X is a martingale), $\mathbb{E}[X_\tau|\mathcal{F}_\sigma] \geq X_\sigma$ (submartingale) and $\mathbb{E}[X_\tau|\mathcal{F}_\sigma] \leq X_\sigma$ (supermartingale) holds respectively.