Definition 0.0.1 (σ-algebra)**.** A family of subsets $\mathcal{A} \subset 2^\Omega$ is called a σ-algebra if it satisfies

(i) $\Omega \in \mathcal{A}$, or $\emptyset \in \mathcal{A}$.

(ii) Closure under complement: If $A \in \mathcal{A}$ then $A^c \in \mathcal{A}$.

(iii) Closure under countable unions and intersections: If a sequence of sets $\{A_i\}_{i\geq 1}$ belongs to $\mathcal{A}$, then $\bigcup_{i=1}^\infty A_i$ (or $\bigcap_{i=1}^\infty A_i$, by De Morgan's law[1]) also belongs to $\mathcal{A}$.

The smallest possible σ-algebra is the trivial σ-algebra: $\{\emptyset, \Omega\}$. The largest possible σ-algebra is 2^Ω. Between the two, we have e.g. for a 6-faced dice, $\mathcal{A} = \{\{1\}, \{2,3,4,5,6\}, \emptyset, \Omega\}$.

[1] $\bigcap_{i=1}^\infty A_i = (\bigcup_{i=1}^\infty A_i^c)^c$.