Lemma 0.0.1. *Assume that M is either an $\mathcal{F}$-submartingale or an $\mathcal{F}$-supermartingale. Then M is an $\mathcal{F}$-martingale if and only if the expected value of M is constant, i.e. $\mathbb{E}[M_0] = \mathbb{E}[M_t]$ for all $t \in [0,T]$.*