Lemma 0.0.1 (quadratic variation of k-dimensional Ito processes)**.** *For $X = (X^1, \ldots, X^k)$ defined in Definition 4.4.2, for any $t \in [0, T]$,*

$$\langle X^i, X^j \rangle_t = \int_0^t \beta_u^i \cdot \beta_u^j \, du = \int_0^t \sum_{l=1}^k \beta_u^{il} \beta_u^{jl} \, du.$$