n

(i) (Novikov's condition) If

$$\mathbb{E}^{\mathbb{P}}\left[\exp\left(\frac{1}{2}\int_0^T |\gamma_u|^2\,du\right)\right] < \infty,$$

then $\mathbb{E}^{\mathbb{P}}(\mathcal{E}_T(U)) = 1$. Consequently, the process $\eta = \mathcal{E}(U)$ is a strictly positive continuous $\mathcal{F}$-martingale. In particular, if the process γ is uniformly bounded, that is, there exists a constant K such that $|\gamma_t| \leq K$ for $t \in [0, T]$, then Novikov's condition is satisfied and thus $\mathbb{E}^{\mathbb{P}}(\mathcal{E}_T(U)) = 1$.

(ii) (Kazamaki's condition) A weaker but also sufficient condition is the Kazamaki condition:

$$\mathbb{E}^{\mathbb{P}}\left[\exp\left(\frac{1}{2}\int_0^t \gamma_u \cdot d\boldsymbol{W}_u\right)\right] < \infty, \quad \forall t \in [0, T].$$

The next result