Definition 0.0.1 (random variable)**.** A **random variable** is a real-valued function $X : \Omega \to \mathbb{R}$ that is measurable with respect to $\mathcal{A}$ and $\mathcal{B}(\mathbb{R})$. That is, the pre-image $X^{-1}(B) = \{\omega : X(\omega) \in B\} \in \mathcal{A}$ for all Borel sets $B \in \mathcal{B}(\mathbb{R})$. We sometimes simply say X is $\mathcal{A}$-measurable.