operties of Brownian motion] Let W be an $\mathcal{F}$-Brownian motion. Then

1. **(martingale property)** W is a continuous $\mathcal{F}$-martingale.
2. **(Markov property)** For any bounded Borel measurable function $g : \mathbb{R} \to \mathbb{R}$ and $0 \leq u \leq t \leq T$,
$$\mathbb{E}[g(W_t)|\mathcal{F}_u] = \mathbb{E}[g(W_t)|\sigma(W_u)] = \underbrace{h(W_u)}_{\text{Doob's measurability theorem}}$$
3. W is a Gaussian process.
4. $\mathrm{Cov}(W_s, W_t) = \mathbb{E}[W_s W_t] = \min(s,t)$ for all $s, t \geq 0$ and in particular $\mathbb{E}[W_t^2] = t$.

Proof. Fo