

Lemma 0.0.1 (Doob's measurability theorem). *Let $X : \Omega \rightarrow \Psi$ be a mapping and (Ψ, \mathcal{G}) a measurable space. A function $Y : \Omega \rightarrow \mathbb{R}$ is $\sigma(X)$ -measurable if and only if there exists a \mathcal{G} -measurable function $h : \Psi \rightarrow \mathbb{R}$ s.t. $Y = h(X)$.*